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CLASS 12

4. INVERSE TRIGONOMETRIC FUNCTIONS

Type I:	Basic, $\sin x, \cos x \to$ sums
Type II:	Find the domain of $\sin^{-1} x, \cos^{-1} x, \tan^{-1} x$
Type III:	Find the principal value of the following (Evaluate)
Type IV:	Find the value using $\sin(A+B), \cos(A+B), \tan(A+B)$
Type V:	Right angled triangle-based sums
Type VI:	(i) $\tan^{-1}(x) + \tan^{-1}(y) = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$ (ii) $\tan^{-1}(x) - \tan^{-1}(y) = \tan^{-1}\left(\frac{x-y}{1+xy}\right)$ – based sums
Type VII:	$\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$ based sums
Type VIII:	$2\tan^{-1} x = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$ $= \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ $= \tan^{-1}\left(\frac{2x}{1-x^2}\right)$ based sums **Total: 101 sums**

4. INVERSE TRIGONOMETRIC FUNCTIONS

Type I: Basic $\sin x, \cos x$ sums

Exercise 4.1

1. Find all the values of x such that

   (i) $-10\pi \le x \le 10\pi$ and $\sin x = 0$ (ii) $-3\pi \le x \le 3\pi$ and $\sin x = -1$

2. Find the period and amplitude of

   (i) $y = \sin 7x$ (ii) $y = -\sin\left(\frac{1}{3}x\right)$ (iii) $y = 4\sin(-2x)$

3. Sketch the graph of $y = \sin\left(\frac{1}{3}x\right)$ for $0 \le x \le 6\pi$.

Exercise 4.2

1. Find all the values of x such that

   (i) $-6\pi \le x \le 6\pi$ and $\cos x = 0$ (ii) $-5\pi \le x \le 5\pi$ and $\cos x = 1$

Type II: Find the domain

Exercise 4.1

6. Find the domain of the following

   (i) $f(x) = \sin^{-1}\left(\frac{x^2+1}{2x}\right)$ (ii) $g(x) = 2\sin^{-1}(2x-1) - \frac{\pi}{4}$

Exercise 4.2

6. Find the domain of

   (i) $f(x) = \sin^{-1}\left(\frac{|x|-2}{3}\right) + \cos^{-1}\left(\frac{1-|x|}{4}\right)$ (ii) $g(x) = \sin^{-1}x + \cos^{-1}x$

Example 4.4

Find the domain of $\sin^{-1}(2-3x^2)$

Exercise 4.3

1. Find the domain of the following functions:

   (i) $\tan^{-1}(\sqrt{9-x^2})$ (ii) $\frac{1}{2}\tan^{-1}(1-x^2) - \frac{\pi}{4}$

Example 4.7

Find the domain of $\cos^{-1}\left(\frac{2+\sin x}{3}\right)$

7. For what value of x, the inequality $\frac{\pi}{2} < \cos^{-1}(3x-1) < \pi$ holds?

Type III: Find the principal value of the following

Exercise 4.1

Note:

$y = \sin^{-1} x$ then $y \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

$y = \cos^{-1} x$ then $y \in [0, \pi]$

$y = \tan^{-1} x$ then $y \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

- $\sin(-\theta) = -\sin\theta$
- $\tan(-\theta) = -\tan\theta$
- $\cos(-\theta) = \cos\theta$
- $-\cos\theta = \cos(\pi - \theta)$

4. Find the value of

   (i) $\sin^{-1}\left(\sin\left(\frac{2\pi}{3}\right)\right)$ (ii) $\sin^{-1}\left(\sin\left(\frac{5\pi}{4}\right)\right)$

Example 4.1

Find the value of $\sin^{-1}\left(\frac{-1}{2}\right)$ (in radians and degree)

Example 4.2

Find the value of $\sin^{-1}(2)$, if it exists.

Example 4.3

Find the principal value of

(i) $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$ (ii) $\sin^{-1}\left(\sin\left(\frac{-\pi}{3}\right)\right)$ (iii) $\sin^{-1}\left(\sin\left(\frac{5\pi}{6}\right)\right)$

Exercise 4.2

2. State the reason for $\cos^{-1}\left[\cos\left(\frac{-\pi}{6}\right)\right] \neq \frac{-\pi}{6}$
3. Is $\cos^{-1}(-x) = \pi - \cos^{-1}(x)$ true? Justify your answer.
4. Find the principal value of $\cos^{-1}\left(\frac{1}{2}\right)$
5. Find the value of

   (i) $2\cos^{-1}\left(\frac{1}{2}\right) + \sin^{-1}\left(\frac{1}{2}\right)$ (ii) $\cos^{-1}\left(\frac{1}{2}\right) + \sin^{-1}(-1)$

8. Find the value of (ii) $\cos^{-1}\left[\cos\frac{4\pi}{3}\right] + \cos^{-1}\left[\cos\frac{5\pi}{4}\right]$

Example 4.5

Find the principal value of $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

Example 4.6

Find (i) $\cos^{-1}\left(\frac{-1}{2}\right)$ (ii) $\cos^{-1}\left[\cos\left(\frac{-\pi}{3}\right)\right]$ (iii) $\cos^{-1}\left[\cos\left(\frac{7\pi}{6}\right)\right]$

Exercise 4.3

2. Find the value of

   (i) $\tan^{-1}\left(\tan\frac{5\pi}{4}\right)$ (ii) $\tan^{-1}\left(\tan\left(\frac{-\pi}{6}\right)\right)$

3. Find the value of

   (i) $\tan\left(\tan^{-1}\left(\frac{7\pi}{4}\right)\right)$ (ii) $\tan(\tan^{-1}(1947))$ (iii) $\tan[\tan^{-1}(-0.0201)]$

Example 4.9

Find (i) $\tan^{-1}(-\sqrt{3})$ (ii) $\tan^{-1}\left(\tan\frac{3\pi}{5}\right)$ (iii) $\tan(\tan^{-1}(2019))$

Example 4.8

Find the principle value of $\tan^{-1}(\sqrt{3})$

4. Find the value of (i) $\tan\left[\cos^{-1}\left(\frac{1}{2}\right)-\sin^{-1}\left(\frac{-1}{2}\right)\right]$

Example 4.10

Find the value of $\tan^{-1}(-1)+\cos^{-1}\left(\frac{1}{2}\right)+\sin^{-1}\left(\frac{-1}{2}\right)$

Exercise 4.4

1. Find the principal value of

   (i) $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$ (ii) $\cot^{-1}(\sqrt{3})$

Example 4.12

Find the principal value of (i) $\operatorname{cosec}^{-1}(-1)$ (ii) $\sec^{-1}(-2)$

Example 4.13

Find the value of $\sec^{-1}\left(\frac{-2\sqrt{3}}{3}\right)$

2. Find the value of (i) $\tan^{-1}(\sqrt{3})-\sec^{-1}(-2)$

   (ii) $\sin^{-1}(-1)+\cos^{-1}\left(\frac{1}{2}\right)+\cot^{-1}(2)$ (iii) $\cot^{-1}(1)+\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right)-\sec^{-1}(-\sqrt{2})$

Exercise 4.5

1. Find the value if it exists. If not give the reason for non-existence

   (i) $\sin^{-1}(\cos\pi)$ (ii) $\tan^{-1}\left[\sin\left(\frac{-5\pi}{2}\right)\right]$ (iii) $\sin^{-1}(\sin 5)$

Exercise 4.17

Simplify: (i) $\cos^{-1}\left[\cos\left(\frac{13\pi}{3}\right)\right]$ (ii) $\tan^{-1}\left(\frac{3\pi}{4}\right)$ (iii) $\sec^{-1}\left[\sec\left(\frac{5\pi}{3}\right)\right]$ (iv) $\sin^{-1}(\sin 10)$

Example 4.18

Find the value of (i) $\sin\left[\frac{\pi}{3}-\sin^{-1}\left(\frac{-1}{2}\right)\right]$

3. Find the of (i) $\sin^{-1}\left[\cos\left(\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)\right]$

Type IV: Find the value using sin (*A* + *B*), cos (*A* + *B*), tan (*A* + *B*)

Exercise 4.1

7. Find the value of $\sin^{-1}\left[\sin\frac{5\pi}{9}\cos\frac{\pi}{9}+\cos\frac{5\pi}{9}\sin\frac{\pi}{9}\right]$

Exercise 4.2

5. Find the value of (iii) $\cos^{-1}\left[\cos\frac{\pi}{7}\cos\frac{\pi}{17}-\sin\frac{\pi}{7}\sin\frac{\pi}{17}\right]$

Exercise 4.3

4. Find the value of

   (ii) $\sin\left[\tan^{-1}\left(\frac{1}{2}\right)-\cos^{-1}\left(\frac{4}{5}\right)\right]$ (iii) $\cos\left[\sin^{-1}\left(\frac{4}{5}\right)-\tan^{-1}\left(\frac{3}{4}\right)\right]$

Exercise 4.5

3. Find the value of (iii) $\tan\left[\sin^{-1}\frac{3}{5}+\cot^{-1}\frac{3}{2}\right]$

4. Prove that (ii) $\sin^{-1}\frac{3}{5}-\cos^{-1}\frac{12}{13}=\sin^{-1}\frac{16}{65}$

Example 4.20

Evaluate $\sin\left[\sin^{-1}\left(\frac{3}{5}\right)+\sec^{-1}\left(\frac{5}{4}\right)\right]$

Example 4.22

If $\cos^{-1}x+\cos^{-1}y+\cos^{-1}z=\pi$ and $0<x, y, z<1$, show that $x^2+y^2+z^2+2xyz=1$.

Type V: Right angled triangle - based sums

Exercise 4.3

Example 4.11

Prove that $\tan(\sin^{-1} x) = \frac{x}{\sqrt{1-x^2}}, -1 < x < 1$

Exercise 4.4

Example 4.14

If $\cot^{-1}\left(\frac{1}{7}\right) = \theta$, Find the value of $\cos\theta$

Example 4.15

Show that $\cot^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right) = \sec^{-1} x, |x| > 1$

Exercise 4.5

Example 4.26

Show that $\cot(\sin^{-1} x) = \frac{\sqrt{1-x^2}}{x}, -1 \le x \le 1$ and $x \ne 0$.

Example 4.19

Prove that $\tan(\sin^{-1} x) = \frac{x}{\sqrt{1-x^2}}$ for $|x| < 1$

Example 4.29

Solve: $\cos\left[\sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)\right] = \sin\left\{\cot^{-1}\left(\frac{3}{4}\right)\right\}$

2. Find the value of the expression in terms of x, with the help of a reference triangle.

   (i) $\sin[\cos^{-1}(1-x)]$ (ii) $\cos[\tan^{-1}(3x-1)]$ (iii) $\tan\left[\sin^{-1}\left(x+\frac{1}{2}\right)\right]$

Type VI: (i) $\tan^{-1}(x) + \tan^{-1}(y) = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$

(ii) $\tan^{-1}(x) - \tan^{-1}(y) = \tan^{-1}\left(\frac{x-y}{1+xy}\right)$ based problems

Exercise 4.5

4. Prove that (i) $\tan^{-1}\frac{2}{11} + \tan^{-1}\frac{7}{24} = \tan^{-1}\frac{1}{2}$.

Example 4.21

Prove that (i) $\tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{1}{3}\right)=\frac{\pi}{4}$

5. Prove that $\tan^{-1}x+\tan^{-1}y+\tan^{-1}z=\tan^{-1}\left(\frac{x+y+z-xyz}{1-xy-yz-zx}\right)$

6. If $\tan^{-1}x+\tan^{-1}y+\tan^{-1}z=\pi$, show that $x+y+z=xyz$

7. Prove that: $\tan^{-1}(x)+\tan^{-1}\left(\frac{2x}{1-x^2}\right)=\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right), |x|<\frac{1}{\sqrt{3}}$

10. Find the number of solution of the equation $\tan^{-1}(x-1)+\tan^{-1}(x)+\tan^{-1}(x+1)=\tan^{-1}(3x)$

Example 4.21

Prove that (ii) $2\tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{1}{7}\right)=\tan^{-1}\left(\frac{31}{17}\right)$

Example 4.27

Solve: $\tan^{-1}(2x)+\tan^{-1}(3x)=\frac{\pi}{4}$ if $6x^2<1$

Example 4.28

Solve: $\tan^{-1}\left(\frac{x-1}{x-2}\right)+\tan^{-1}\left(\frac{x+1}{x+2}\right)=\frac{\pi}{4}$

8. Simplify: $\tan^{-1}\left(\frac{x}{y}\right)-\tan^{-1}\left(\frac{x-y}{x+y}\right)$

Example 4.24

Solve: $\tan^{-1}\left(\frac{1-x}{1+x}\right)=\frac{1}{2}+\tan^{-1}x$ for $x>0$

9. Solve: (iv) $\cot^{-1}(x)-\cot^{-1}(x+2)=\frac{\pi}{12}, x>0.$

Example 4.23

If $a_1, a_2, a_3 \ldots$ is an arithmetic progression with common difference 'd', prove that

$$\tan\left[\tan^{-1}\left(\frac{d}{1+a_1a_2}\right)+\tan^{-1}\left(\frac{d}{1+a_2a_3}\right)+\ldots+\tan^{-1}\left(\frac{d}{1+a_na_{n-1}}\right)\right]=\frac{a_n-a_1}{1+a_1a_n}$$

Type VII: $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$ based sums

Exercise 4.2

8. Find the value of (i) $\cos\left[\cos^{-1}\left(\frac{4}{5}\right) + \sin^{-1}\left(\frac{4}{5}\right)\right]$

Exercise 4.5

3. Find the value of (ii) $\cot\left[\sin^{-1}\frac{3}{5} + \sin^{-1}\frac{4}{5}\right]$

Example 4.16

Prove that $\frac{\pi}{2} \le \sin^{-1} x + 2\cos^{-1} x \le \frac{3\pi}{2}$

Example 4.25

Solve: $\sin^{-1} x > \cos^{-1} x$

Type VIII: $2\tan^{-1} x$ based sums

(i) $2\tan^{-1} x = \tan^{-1}\left(\frac{2x}{1-x^2}\right), |x| < 1$

(ii) $2\tan^{-1} x = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right), x \ge 0$

(iii) $2\tan^{-1} x = \sin^{-1}\left(\frac{2x}{1+x^2}\right), |x| < 1$

Example 4.18

(iii) $\tan\left[\frac{1}{2}\sin^{-1}\left(\frac{2a}{1+a^2}\right) + \frac{1}{2}\cos^{-1}\left(\frac{1-a^2}{1+a^2}\right)\right]$

9. Solve: (ii) $2\tan^{-1} x = \cos^{-1}\left(\frac{1-a^2}{1+a^2}\right) - \cos^{-1}\left(\frac{1-b^2}{1+b^2}\right), a > 0, b > 0$

   (iii) $2\tan^{-1} x(\cos x) = \tan^{-1}(2 \operatorname{cosec} x)$

Example 4.18

(ii) Find the value of $\cos\left[\frac{1}{2}\cos^{-1}\left(\frac{1}{8}\right)\right]$

9. Solve: (i) $\sin^{-1}\left(\frac{5}{x}\right) + \sin^{-1}\left(\frac{12}{x}\right) = \frac{\pi}{2}$

CHAPTER 1. APPLICATIONS OF MATRICES AND DETERMINANTS

UNIT TEST: 1 (Ex 1.1, 1.2)

Time: 40 min **Marks: 25**

I. Answer the following **($3 \times 2 = 6$)**

1. Find the inverse of $\begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

2. Find adj (adj A) if adj $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix}$

3. Find the rank of $\begin{bmatrix} 1 & -2 & 3 \\ 2 & 4 & -6 \\ 5 & 1 & -1 \end{bmatrix}$ by minor method.

II. Answer the following **($3 \times 3 = 9$)**

4. If $A = \begin{bmatrix} 8 & -4 \\ -5 & 3 \end{bmatrix}$ verify that $A\,(\text{adj}\,A) = (\text{adj}\,A)\,A = |A|\,I_2$.

5. If $A = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$ show that $A^2 - 3A - 7I_2 = 0_2$ Hence find A^{-1}.

6. Find the rank of $\begin{bmatrix} 1 & -2 & 3 & 4 \\ -2 & 4 & -1 & -3 \\ -1 & 2 & 7 & 6 \end{bmatrix}$

III. Answer the following **($2 \times 5 = 10$)**

7. If $A = \begin{bmatrix} -8 & 1 & 4 \\ 4 & 4 & 7 \\ 1 & -8 & 4 \end{bmatrix}$ prove that $A^{-1} = A^T$

8. Find the inverse of $A = \begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}$ by Gauss-Jordan method.

UNIT TEST: 2 (Ex 1.3, 1.4)

Time: 40 min **Marks: 25**

I. Answer the following **$(2 \times 2 = 4)$**

1. Solve he following system of linear equations by matrix inversion method $2x - y = 8$; $3x + 2y = -2$
2. Solve the following system of linear equations by Cramer's rule $5x - 2y + 16 = 0$; $x + 3y - 7 = 0$.

II. Answer the following **$(2 \times 3 = 6)$**

3. Solve $\frac{3}{x} + 2y = 12$; $\frac{2}{x} + 3y = 13$ by matrix inversion method.
4. In a competitive examination, one mark is awarded for every correct answer while $\frac{1}{4}$ mark is deduced for every wrong answer. A student answered 100 Questions and got 80 marks. How many Questions did he answer correctly? (use Cramer's) rule to solve the problem).

III. Answer the following **$(3 \times 5 = 15)$**

5. Solve: $\frac{1}{x} + \frac{2}{y} - \frac{1}{z} = 1$; $\frac{2}{x} + \frac{4}{y} + \frac{1}{z} = 5$; $\frac{3}{x} - \frac{2}{y} - \frac{2}{z} = 0$ using cramer's rule.
6. If $A = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ find the products AB and BA and hence solve the system of equations $x - y + z = 4$; $x - 2y - 2z = 9$; $2x + y + 3z = 1$.
7. In a T-20 match, Chennai super kings needed just 6 runs to win with 1 ball left to go in the last over. The last ball was bowled and the batsman at the crease hit it high up. The ball traversed along a path in a vertical plane and the equation of the path is $y = ax^2 + bx + c$ with respect to xy co-ordinate system in the vertical plane and the ball traversed through the point (10,8), (20,16), (30,18). Can you conclude that Chennai super kings won the match? Justify your answer. (All the distances are measured in metres and the meeting point of the plane of the path with the farthest boundary line is (70, 0).

UNIT TEST: 1 [Ex 1.5, 1.6, 1.7]

Time: 40 min **Marks: 25**

I. Answer the following **$(5 \times 5 = 25)$**

1. Solve the following system of linear equations by Gaussian elimination method $2x - 2y + 3z = 2$; $x + 2y - z = 3$; $3x - y + 2z = 1$
2. Test for consistency of the following system of linear equations can if possible solve: $x + 2y + z = 6$; $3x + 3y - z = 3$; $2x + y - 2z = -3$.
3. Find the value of k for which the equation $kx - 2y + z = 1$, $x - 2ky + z = -2$, $x - 2y + kz = 1$ have (i) no solution (ii) unique solution (ii) infinitely many solution.
4. By using Gaussian elimination method balance the chemical reaction equation $C_2H_6 + O_2 \longrightarrow H_2O + CO_2$
5. An amount 65,000 is inverted in three bonds at the rates of 6%, 8% and 10% per annum respectively. The total annual income is 5000. The income from the third bond is Rs.600 more than that from the second bond. Determine the price of each bond (use Gaussian elimination method).

CHAPTER TEST 1: APPLICATIONS OF MATRICES AND DETERMINANTS

Time: 1.30 min **Marks: 50**

I. Choose the correct answer **$(10 \times 1 = 10)$**

1. If $|\text{adj}\,(\text{adj}\, A| = |A|^9$, then the order of the square matrix A is

 (A) 3 (B) 4 (C) 2 (D) 5

2. If the rank of the matrix $\begin{bmatrix} \lambda & -1 & 0 \\ 0 & \lambda & -1 \\ -1 & 0 & \lambda \end{bmatrix}$ is 2 then λ is

 (A) 1 (B) 2 (C) 3 (D) any real number

3. If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ be such that $\lambda A^{-1} = A$, then λ is

 (A) 17 (B) 14 (C) 19 (D) 21

4. If A, B and C are invertible matrices of same order, then which one of the following is not true?

 (A) $\text{adj}\, A = |A|\, A^{-1}$ (B) $\text{adj}\,(AB) = (\text{adj}\, A)\,(\text{adj}\, B)$

 (C) $\det A^{-1} = (\det A)^{-1}$ (D) $(ABC)^{-1} = C^{-1} B^{-1} A^{-1}$

5. If $A = [2 \quad 0 \quad 1]$ then rank of AA^T is

 (A) 1 (B) 2 (C) 3 (D) 0

6. If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is non-singular then $A^{-1} =$

 (A) $\dfrac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ (B) $\dfrac{1}{bc - ad}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

 (C) $\dfrac{1}{ad - bc}\begin{bmatrix} -d & b \\ c & -a \end{bmatrix}$ (D) $\dfrac{1}{bc - ad}\begin{bmatrix} a & -b \\ -c & d \end{bmatrix}$

7. If $A = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$ then $9I - A =$

 (A) A^{-1} (B) $\dfrac{A^{-1}}{2}$ (C) $3A^{-1}$ (D) $2A^{-1}$

8. The augmented matrix of a system of linear equations is $\begin{bmatrix} 1 & 2 & 7 & 3 \\ 0 & 1 & 4 & 6 \\ 0 & 0 & \lambda - 4 & \mu + 5 \end{bmatrix}$. The system has infinitely many solutions if

 (A) $\lambda = 7, \mu \neq -5$ (B) $\lambda = -7, \mu = 5$

 (C) $\lambda \neq 7, \mu \neq -5$ (D) $\lambda = 7, \mu = -5$

9. If A is a matrix of order 3, then det (kA)

(A) k^3 det (A) (B) k^2 det (A) (C) k det (A) (D) det (A)

10. If $ae^x + be^y = c$; $pe^x + qe^y = d$ and $\Delta_1 = \begin{vmatrix} a & b \\ p & q \end{vmatrix}$; $\Delta_2 = \begin{vmatrix} c & b \\ d & q \end{vmatrix}$; $\Delta_3 = \begin{vmatrix} a & c \\ p & d \end{vmatrix}$ then the value of (x, y) is

(A) $\left(\frac{\Delta_2}{\Delta_1}, \frac{\Delta_3}{\Delta_1}\right)$ (B) $\left(\log\frac{\Delta_2}{\Delta_1}, \log\frac{\Delta_3}{\Delta_1}\right)$

(C) $\left(\log\frac{\Delta_1}{\Delta_3}, \log\frac{\Delta_1}{\Delta_2}\right)$ (D) $\left(\log\frac{\Delta_1}{\Delta_2}, \log\frac{\Delta_1}{\Delta_3}\right)$

II. Answer any 3 of the following (Q.No: 14 is Compulsory) **[$3 \times 2 = 6$]**

11. If $A = \begin{bmatrix} 0 & -3 \\ 1 & 4 \end{bmatrix}$, $B = \begin{bmatrix} -2 & -3 \\ 0 & -1 \end{bmatrix}$ find $(AB)^{-1}$.

12. If adj $(A) = \begin{bmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 0 \end{bmatrix}$ Find A^{-1}.

13. Find the rank of the matrix $\begin{bmatrix} 1 & 1 & 1 & 3 \\ 2 & -1 & 3 & 4 \\ 5 & -1 & 7 & 11 \end{bmatrix}$ by reducing into an echelon form.

14. If the augmented matrix is $[A \mid 0]$ is $\begin{bmatrix} 1 & 1 & 3 & 0 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & \lambda - 8 & 0 \end{bmatrix}$ then determine the value of λ for

(i) unique solution (ii) a non-trivial solution

III. Answer any 3 of the following (Q.No: 18 is Compulsory) **[$3 \times 3 = 9$]**

15. If $A = \begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$ show that $A^T \cdot A^{-1} = \begin{bmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{bmatrix}$

16. Solve by matrix inversion method: $7x + 3y + 1 = 0$; $2x + y = 0$

17. Determine the values of λ for which the following system of equations $(3\lambda - 8)\,x + 3y + 3z = 0$; $3x + (3\lambda - 8)\,y + 3z = 0$; $3x + 3y + (3\lambda - 8)\,z = 0$ has a non-trivial solution.

18. A fish tank can be filled in 10 minutes using both pumps A and B simultaneously. However, pump B can pump water in or out at the same rate. If pump B is inadvertently run in reverse, then the tank will be filled in 30 minutes. How long would it take each pump to fill the tank by itself? (use Cramer's rule to solve the problem).

IV. Answer the following **($5 \times 5 = 25$)**

19. If $F(\alpha) = \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix}$ show that $[F(\alpha)]^{-1} = F(-\alpha)$.

20. Solve the following system of equation using matrix inversion method: $2x_1 + 3x_2 + 3x_3 = 5$; $x_1 - 2x_2 + x_3 = -4$; $3x_1 - x_2 - 2x_3 = 3$.

21. If $ax^2 + bx + c$ is divided by $x + 3$, $x - 5$ and $x - 1$ the remainders are 21, 61 and 9 respectively. Find a, b and c (use Gaussian elimination method).

22. Test for consistency and if possible solve the following system of equations by rank method.

$$3x + y + z = 2 \ ; \ x - 3y + 2z = 1 \ ; \ 7x - y + 4z = 5$$

23. Investigate for what values of λ and μ the system of linear equations $x + 2y + z = 7$, $x + y + \lambda z = \mu$, $x + 3y - 5z = 5$ has (i) no solution (ii) a unique solutions (iii) an infinite number of solutions

24. By using Gaussian elimination method, balance the chemical reaction equation
$C_2H_8 + O_2 \longrightarrow CO_2 + H_2O$

25. If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$ verify A (adj A) = (adj A) $A = |A|\, I_3$

CHAPTER 2. COMPLEX NUMBERS

UNIT TEST: 1 (Ex 2.1, 2.2, 2.3, 2.4)

Time: 40 min **Marks: 25**

I. Answer the following **($4 \times 2 = 8$)**

1. Simplify: $\sum_{n=1}^{102} i^n$
2. If $z = -3 - 4i$ find multiplicative inverse of z.
3. Find the least positive integer n such that $\left(\frac{1+i}{1-i}\right)^n = 1$.
4. Prove the following property: z is purely imaginary if and only if $z = -\bar{z}$

II. Answer the following **($4 \times 3 = 12$)**

5. Find the values of the real numbers x and y, if the complex numbers $(3 - i)x - (2 - i)y + 2i + 5$ and $2x + (-1 + 2i)y + 3 + 2i$ are equal.
6. If $z_1 = 1 - 3i$, $z_2 = -4i$ and $z_3 = 5$ show that $(z_1 z_2) z_3 = z_1 (z_2 z_3)$
7. The complex numbers u, v and w are related by $\frac{1}{u} = \frac{1}{v} + \frac{1}{w}$ if $v = 3 - 4i$ and $w = 4 + 3i$ find u in rectangular form.
8. If $\frac{z+3}{z-5i} = \frac{1+4i}{2}$ find the complex number z.

III. Answer the following **($1 \times 5 = 5$)**

9. Show that (i) $(2 + i\sqrt{3})^{10} - (2 - i\sqrt{3})^{10}$ is purely imaginary

   (ii) $\left(\frac{19-7i}{9+i}\right)^{12} + \left(\frac{20-5i}{7-6i}\right)^{12}$ is real.

UNIT TEST: 2 (Ex 2.5, 2.6)

I. Answer the following **$(3 \times 2 = 6)$**

1. Find the following of $(1-i)^{10}$.
2. Show that the equation $z^2 = \overline{z}$ has four solutions.
3. Show that $|3z - 5 + i| = 4$ represents a circle and find its centre and radius.

II. Answer the following **$(3 \times 3 = 9)$**

4. Find the square root of $6 - 8i$
5. Show that the points representing the complex numbers $7 + 9i, -3 + 7i, 3 + 3i$ form a right angled triangle on the Argand diagram.
6. Obtain the cartesian equation for the locus of $z = x + iy$ in $|z + i| = |z - i|$.

III. Answer the following **$(2 \times 5 = 10)$**

7. If $z = x + iy$ is a complex number such that $Im\left(\frac{2z+1}{iz+1}\right) = 0$, show that the locus of z is $2x^2 + 2y^2 + x - 2y = 0$.
8. Let $z_1\ z_2$ and z_3 be complex numbers such that $|z_1| = |z_2| = |z_3| = r > 0$ and $z_1 + z_2 + z_3 \neq 0$ prove that $\left|\frac{z_1 z_2 + z_2 z_3 + z_3 z_1}{z_1 + z_2 + z_3}\right| = r$.

UNIT TEST: 3 [Ex 2.7, 2.8]

I. Answer the following **$(2 \times 2 = 4)$**

1. Simplify: $\left(\dfrac{1 + \sin\theta + i\cos\theta}{1 + \sin\theta - i\cos\theta}\right)^n$

2. If $\omega \neq 1$ is a cube root of unity show that $(1 - \omega + \omega^2)^6 + (1 + \omega - \omega^2)^6 = 128$.

II. Answer the following **$(2 \times 3 = 6)$**

3. Solve the equation $z^3 + 27 = 0$

4. Write in polar form: $\dfrac{i - 1}{\cos\dfrac{\pi}{3} + i\sin\dfrac{\pi}{3}}$.

III. Answer the following **$(3 \times 5 = 15)$**

5. If $z = x + iy$ and $\arg\left(\dfrac{z - 1}{z + 1}\right) = \dfrac{\pi}{2}$ then show that $x^2 + y^2 = 1$

6. If $2\cos\alpha = x + \dfrac{1}{x}$ and $2\cos\beta = y + \dfrac{1}{y}$ show that

   (i) $\dfrac{x^m}{y^n} + \dfrac{y^n}{x^m} = 2\cos(m\alpha - n\beta)$

   (ii) $xy - \dfrac{1}{xy} = 2i\sin(\alpha + \beta)$

7. Suppose z_1, z_2 and z_3 are the vertices of an equilateral triangle inscribed in the circle $|z| = 2$. If $z_1 = 1 + i\sqrt{3}$ then find z_2 and z_3.

CHAPTER TEST 2. COMPLEX NUMBERS

Time: 1.30 HRS **Marks: 50**

I. Choose the correct answer **($10 \times 1 = 10$)**

1. $i^n + i^{n+1} + i^{n+2} + i^{n+3}$ is

   (A) 0 (B) 1 (C) -1 (D) i

2. If $\left| z - \frac{3}{z} \right| = 2$, then the least value of $|z|$ is

   (A) 1 (B) 2 (C) 3 (D) 5

3. If $(1+i)(1+2i)(1+3i) \ldots (1+ni) = x + iy$ then $2.5.10 \ldots (1+n^2)$ is

   (A) 1 (B) i (C) $x^2 + y^2$ (D) $1 + n^2$

4. Which of the following one is not true

   (A) $|z_1 + z_2| \leq |z_1| + |z_2|$ (B) $|z_1 - z_2| \leq |z_1| - |z_2|$

   (C) $|z_1 \cdot z_2| = |z_1|\,|z_2|$ (D) $\left| \frac{z_1}{z_2} \right| = \frac{|z_1|}{|z_2|}, |z_2| \neq 0$

5. If z represents a complex number then $\arg(z) + \arg(\bar{z})$ is

   (A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$ (C) 0 (D) $\frac{\pi}{3}$

6. $z_1 = 4 + 5i$, $z_2 = -3 + 2i$ then $\frac{z_1}{z_2}$ is

   (A) $\frac{2}{13} - \frac{22}{13}i$ (B) $\frac{-2}{13} + \frac{22}{13}i$ (C) $\frac{-2}{13} - \frac{23}{13}i$ (D) $\frac{2}{13} + \frac{22}{13}i$

7. The value of $\left[\frac{-1 + i\sqrt{3}}{2} \right]^{100} + \left[\frac{-1 - i\sqrt{3}}{2} \right]$ is

   (A) 2 (B) 0 (C) -1 (D) 1

8. The value of $\left(\frac{1 + \sqrt{3}i}{1 - \sqrt{3}i} \right)^{10}$ is

   (A) $cis\,\frac{2\pi}{3}$ (B) $cis\,\frac{4\pi}{3}$ (C) $-cis\,\frac{2\pi}{3}$ (D) $-cis\,\frac{4\pi}{3}$

9. If $|z_1| = 1$, $|z_2| = 2$, $|z_3| = 3$ and $|9z_1 z_2 + 4z_1 z_3 + z_2 z_3| = 12$ then the value of $|z_1 + z_2 + z_3|$ is

   (A) 1 (B) 2 (C) 3 (D) 4

10. If $x = \cos\theta + i\sin\theta$ the value of $x^n + \frac{1}{x^n}$ is

    (A) $2\cos n\theta$ (B) $2i\sin n\theta$ (C) $2\sin n\theta$ (D) $2i\cos n\theta$

II. Answer any 4 of the following (Q.No: 15 is Compulsory) **[$4 \times 2 = 8$]**

11. Simplify: $i i^2 i^3 \ldots i^{40}$

12. Simplify: $\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3$

13. If $|z| = 1$ show that $2 \le |z^2 - 3| \le 4$

14. If $(x_1 + iy_1)(x_2 + iy_2)(x_3 + iy_3) \ldots (x_n + iy_n) = a + ib$ show that
$(x_1^2 + y_1^2)(x_2^2 + y_2)^2 (x_3^2 + y_3)^2) \ldots (x_n^2 + y_n^2) = a^2 + b^2$

15. Simplify: $\left(\sin\frac{\pi}{6} + i\cos\frac{\pi}{6}\right)^{18}$

III. Answer any 4 of the following (Q.No: 20 is Compulsory) **[$4 \times 3 = 12$]**

16. Show that the points $1, \frac{-1}{2} + \frac{i\sqrt{3}}{2}$ and $\frac{-1}{2} - \frac{i\sqrt{3}}{2}$ are the vertices of an equilateral triangle.

17. Find the square root of $-5 - 12i$

18. Find the rectangular form of $\dfrac{\cos\frac{\pi}{6} - i\sin\frac{\pi}{6}}{2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)}$

19. If n is a positive integer, prove that $(\sqrt{3} + i)^n + (\sqrt{3} - i)^n = 2^{n+1}\cos\frac{n\pi}{6}$.

20. State and prove "Triangle inequality".

IV. Answer any 4 of the following **[$4 \times 5 = 20$]**

21. P represents the variable complex number z, find the locus of P is $\text{Re}\left(\frac{z+1}{z+i}\right) = 1$.

22. Find all cube roots of $\sqrt{3} + i$

23. Prove that the values of $\sqrt[4]{-1}$ are $\pm\frac{1}{\sqrt{2}}(1 \pm i)$

24. Show that (i) $(2 + i\sqrt{3})^{10} + (2 - i\sqrt{3})^{10}$ is real and (ii) $\left(\frac{19+9i}{5-3i}\right)^{15} - \left(\frac{8+i}{1+2i}\right)^{15}$ is purely imaginary.

25. If $\cos\alpha + \cos\beta + \cos\gamma = 0 = \sin\alpha + \sin\beta + \sin\gamma$ prove that

    (i) $\cos 3\alpha + \cos 3\beta + \cos 3\gamma = 3\cos(\alpha + \beta + \gamma)$

    (ii) $\sin 3\alpha + \sin 3\beta + \sin 3\gamma = 3\sin(\alpha + \beta + \gamma)$

CHAPTER 3. THEORY OF EQUATION

UNIT TEST: 1 (Ex 3.1, 3.2, 3.3)

Time: 40 min **Marks: 25**

I. Answer the following **$(3 \times 2 = 6)$**

1. Construct a cubic equation with roots $2, -2$ and 4.
2. Find a polynomial equation of minimum degree with rational co-efficients having $2i + 3$ as a root.
3. Solve the equation $x^4 - 9x^2 + 20 = 20$.

I. Answer the following **$(3 \times 3 = 9)$**

4. If p and q are the roots of the equation $lx^2 + nx + n = 0$, show that $\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{n}{l}} = 0$.
5. Solve the equation $x^3 - 9x^2 + 14x + 24 = 0$. If it is given that two of its roots are in the ratio 3:2
6. If the roots of $x^3 + px^2 + qx + r = 0$ are in H.P. prove that $9pqr = 27r^2 + 2q^3$

III. Answer the following **$(2 \times 5 = 10)$**

7. If α, β and γ are the roots of the cubic equation $x^3 + 2x^2 + 3x + 4 = 0$, form a cubic equation whose roots are (i) $2\alpha, 2\beta, 2\gamma$ (ii) $\frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\gamma}$ (iii) $-\alpha, -\beta, -\gamma$
8. Find all zeros of the polynomial $x^6 - 3x^5 - 5x^4 + 22x^3 - 39x^2 - 39x + 135$, if it is known that $1 + 2i$ and $\sqrt{3}$ are two of its zeros.

UNIT TEST: 2 [Ex 3.4, 3.5, 3.6]

Time: 40 min **Marks: 25**

I. Answer the following **$(2 \times 2 = 4)$**

1. Solve the equation: $x^3 - 5x^2 - 4x + 20 = 0$
2. Show that the equation $x^9 - 5x^5 + 4x^4 + 2x^2 + 1 = 0$ has atleast 6 imaginary solutions.

II. Answer the following **$(2 \times 3 = 6)$**

3. Find all real numbers satisfying $4^x - 3\,(2^{x+2}) + 2^5 = 0$
4. Solve the equations $6x^4 - 5x^3 - 38x^2 - 5x + 6 = 0$ if it known that 1/3 is a solution.

III. Answer the following **$(3 \times 5 = 15)$**

5. Solve $(x-5)\,(x-7)\,(x+6)\,(x+4) = 504$
6. Solve the equation: $6x^4 - 35x^3 + 62x^2 - 35x + 6 = 0.$
7. Examine for the rational roots of (i) $2x^3 - x^2 - 1 = 0$ (ii) $x^8 - 3x + 1 = 0$

CHAPTER TEST 3. THEORY OF EQUATION
(Full) - TEST

Time: 1.30 hrs **Marks: 50**

I. Choose the correct answer **$(10 \times 1 = 10)$**

1. A zero of $x^3 + 64$ is

   (A) 0 (B) 2 (C) $4i$ (D) -4

2. A polynomial equation in x of degree n always has

   (A) n distinct roots (B) n real roots
   (C) n imaginary roots (D) at most one root

3. If $x^2 + 2(k+2)x + 9k = 0$ has equal roots then the value of k are

   (A) 1 and 4 (B) -1 and 4 (C) -1 and -4 (D) 1 and -4

4. If α, β and γ are the roots of the equation $x^3 + px^2 + qx + r = 0$ then the value of $\Sigma \frac{1}{\beta r}$ is

   (A) $\frac{p}{q}$ (B) $\frac{p}{r}$ (C) $\frac{-p}{r}$ (D) $\frac{-p}{q}$

5. According to the rational root theorem, which number is not possible rational root of $4x^7 + 2x^4 - 10x^3 - 5$.

   (A) -1 (B) $\frac{5}{4}$ (C) $\frac{4}{5}$ (D) 5

6. The number of real number in $[0, 2\pi]$ satisfying $\sin^4 x - 2\sin^2 x + 1$ is

   (A) 2 (B) 4 (C) 1 (D) ∞

7. If $2i - \sqrt{3}$ is one root of a polynomial equation, then another root is

   (A) $2i + \sqrt{3}$ (B) $-2i + \sqrt{3}$ (C) $-\sqrt{3} - 2i$ (D) $\sqrt{3}$

8. The nature of roots of the equation $x^{2018} + 1947x^{1950} + 15x^8 + 26x^6 + 2019$ is

   (A) real and imaginary roots (B) only real roots
   (C) only imaginary roots (D) '0' only

9. If $-i + 3$ is a root of $x^2 - 6x + k = 0$, then the value of k is

   (A) 5 (B) $\sqrt{5}$ (C) $\sqrt{10}$ (D) 10

10. The number of positive roots of the polynomial $\sum_{i=0}^{n} nC_r (-1)^r x^r$ is

    (A) 0 (B) n (C) $< n$ (D) r

II. Answer any 4 of the following (Q.No: 15 is Compulsory) **[$4 \times 2 = 8$]**

11. Formulate into a mathematical problem to find a number such that when its cube root is added to if the result is 6.

12. Show that if p, q, r are rational, the roots of the equation $x^2 - 2px + p^2 - q^2 + 2qr - r^2 = 0$ are rational.

13. Solve the equation $x^3 - 3x^2 - 33x + 35 = 0$

14. Determine the number of positive and negative roots of the equation $x^9 - 5x^8 - 14x^7 = 0$.

15. Prove that a line cannot intersect a circle at more than two points.

III. Answer any 4 of the following (Q.No: 20 is Compulsory) **[$4 \times 3 = 12$]**

16. Solve: $8x^{3/2n} - 8x^{-3/2n} = 63$

17. It is known that the roots of the equation $x^3 - 6x^2 - 4x + 24 = 0$ are in A.P. Find its roots.

18. Solve the cubic equation $2x^3 - x^2 - 18x + 9 = 0$ if sum of its two roots vanishes.

19. If p is real, discuss the nature of the roots of the equation $4x^2 + 4px + p + 2 = 0$ in terms of P.

20. Form a polynomial equation with integer coefficients with $\sqrt{\frac{\sqrt{2}}{\sqrt{3}}}$ as a root.

IV. Answer any 4 of the following **($4 \times 5 = 20$)**

21. If the equations $x^2 + px + q = 0$ and $x^2 + p'x + q' = 0$ have a common root, show that it be equal to $\frac{pq' - p'q}{q - q'}$ or $\frac{q - q'}{p' - p}$

22. If $2 + i$ and $3 - \sqrt{2}$ are roots of the equation $x^6 - 13x^5 + 62x^4 - 126x^3 + 65x^2 + 127x - 140 = 0$ Find all zeros.

23. Solve: $(2x - 1)(x + 3)(x - 2)(2x + 3) + 20 = 0$

24. Solve the following equation $x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$.

25. Solve: $2\sqrt{\frac{x}{a}} + 3\sqrt{\frac{a}{x}} = \frac{b}{a} + \frac{6a}{b}$

CHAPTER 4. INVERSE TRIGONOMETRIC FUNCTIONS

UNIT TEST: I (Ex 4.1, 4.2, 4.3, 4.4)

Time: 40 min **Marks: 25**

I. Answer the following **$(3 \times 2 = 6)$**

1. Find the period and amplitude of $y = -\sin\left(\frac{1}{3}x\right)$
2. Evaluate: $\cos^{-1}\left(\cos\frac{\pi}{7}\cos\frac{\pi}{17} - \sin\frac{\pi}{7}\sin\frac{\pi}{17}\right)$
3. Find the value of $\tan^{-1}\left(\tan\frac{5\pi}{4}\right)$

II. Answer the following **$(3 \times 3 = 9)$**

4. Find the value of $\cos\left[\sin^{-1}\left(\frac{4}{5}\right) - \tan^{-1}\left(\frac{3}{4}\right)\right]$
5. Sketch the graph of $y = \sin\left(\frac{1}{3}x\right)$ for $0 \le x < 6\pi$
6. Find the domain of $\cos^{-1}\left(\frac{2 + \sin x}{3}\right)$

III. Answer the following **$(3 \times 5 = 15)$**

7. Find the domain of the following

   (i) $f(x) = \sin^{-1}\left(\frac{x^2 + 1}{2x}\right)$ (ii) $\tan^{-1}(\sqrt{9 - x^2})$
8. Prove that $\tan(\sin^{-1} x) = \frac{x}{\sqrt{1 - x^2}}, -1 < x < 1$
9. Find the value of $\cot^{-1}(1) + \sin^{-1}\left(\frac{-\sqrt{3}}{2}\right) - \sec^{-1}(-\sqrt{2})$

UNIT TEST: II (Ex 4.5)

Time: 40 min **Marks: 25**

I. Answer the following $(3 \times 2 = 6)$

1. Find the value of $\sin^{-1}(\sin 10)$
2. Find the value of the expression in terms of x, with the help of a reference triangle $\sin(\cos^{-1}(1-x))$
3. Prove that $\frac{\pi}{2} \leq \sin^{-1} x + 2\cos^{-1} x \leq \frac{3\pi}{2}$

II. Answer the following $(3 \times 3 = 9)$

4. Show that $2\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{7}\right) = \tan^{-1}\left(\frac{31}{17}\right)$
5. Find the value of $\tan\left(\sin^{-1}\frac{3}{5} + \cot^{-1}\frac{3}{2}\right)$
6. Solve $\sin^{-1}\frac{5}{x} + \sin^{-1}\frac{12}{x} = \frac{\pi}{2}$

III. Answer the following $(3 \times 5 = 15)$

7. Solve: $\cot^{-1} x - \cot^{-1}(x+2) = \frac{\pi}{12}, x > 0$
8. Find the number of solution of the equation

$$\tan^{-1}(x-1) + \tan^{-1} x + \tan^{-1}(x+1) = \tan^{-1}(3x)$$

9. If $\cos^{-1} x + \cos^{-1} z = \pi$ and $0 < x, y, z < 1$ show that $x^2 + y^2 + z^2 + 2xyz = 1$.

CHAPTER TEST 4. INVERSE TRIGONOMETRIC FUNCTIONS
(Full-Test)

Time: 1.30 hrs **Marks: 50**

I. Choose the correct answer **$(10 \times 1 = 10)$**

1. The value of $\sin^{-1}(\cos x), 0 \le x \le \pi$ is

   (A) $\pi - x$ (B) $x - \frac{\pi}{2}$ (C) $\frac{\pi}{2} - x$ (D) $\pi - x$

2. $\sin^{-1}\frac{3}{5} - \cos^{-1}\frac{12}{13} + \sec^{-1}\frac{5}{3} - \text{cosec}^{-1}\frac{13}{12}$ is equal to

   (A) 2π (B) π (C) 0 (D) $\tan^{-1}\frac{12}{65}$

3. Principal value of $\cos^{-1}\left(\cos\frac{5\pi}{3}\right)$

   (A) $\frac{5\pi}{3}$ (B) $\frac{\pi}{3}$ (C) $\frac{-\pi}{3}$ (D) $\frac{2\pi}{3}$

4. Find x if $\tan^{-1} 4 + \cot^{-1} x = \frac{\pi}{2}$

   (A) 4 (B) -4 (C) 1/4 (D) not defined

5. $\sin^{-1}(\cos x) = \pi/2 - x$ is valid for

   (A) $-\pi \le x \le 0$ (B) $0 \le x \le \pi$ (C) $\frac{-\pi}{2} \le x \le \frac{\pi}{2}$ (D) $\frac{-\pi}{4} \le x \le \frac{3\pi}{4}$

6. The domain of the function defined by $f(x) = \sin^{-1}\sqrt{x-1}$ is

   (A) $[-1, 0]$ (B) $[0, 1]$ (C) $[-1, 1]$ (D) $[1, 2]$

7. If $x = \frac{1}{5}$, the value of $\cos(\cos^{-1} x + 2\sin^{-1} x)$ is

   (A) $-\sqrt{\frac{24}{25}}$ (B) $\sqrt{\frac{24}{25}}$ (C) $\frac{1}{5}$ (D) $-\frac{1}{5}$

8. If $\cot^{-1}(2)$ and $\cot^{-1}(3)$ are the two angles of a triangle, then the third angle is

   (A) $\frac{\pi}{4}$ (B) $\frac{3\pi}{4}$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{3}$

9. If $\sin^{-1} x + \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{2}$ then x is

   (A) $\frac{1}{2}$ (B) $\frac{-1}{2}$ (C) 2 (D) -2

10. The equation $\tan^{-1} x - \cot^{-1} x = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ has

    (A) No solution (B) Two solution
    (C) Unique solution (D) infinite number of solutions

II. Answer any 4 of the following (Q.No: 15 is Compulsory) **$(4 \times 2 = 8)$**

11. For what value of x does $\sin x = \sin^{-1} x$.

12. Find all the values of x such that $-5\pi \leq x \leq 5\pi$ and $\cos x = 1$.

13. Find the principal value of $\operatorname{cosec}^{-1}(-\sqrt{2})$.

14. Prove that $\tan^{-1}\left(\frac{2}{11}\right) + \tan^{-1}\left(\frac{7}{24}\right) = \tan^{-1}\left(\frac{1}{2}\right)$

15. Find the if it exists $\sin^{-1}(\sin 5)$

III. Answer any 4 of the following (Q.No: 20 is Compulsory) **$(4 \times 3 = 12)$**

16. Find the domain of $\sin^{-1}(2 - 3x^2)$.

17. Find the value of $\sin\left(\tan^{-1}\left(\frac{1}{2}\right) - \cos^{-1}\left(\frac{4}{5}\right)\right)$

18. Show that $\cot^{-1}\left(\frac{1}{\sqrt{x^2 - 1}}\right) = \sec^{-1} x, |x| > 1$

19. Find the value of $\tan\left[\frac{1}{2}\sin^{-1}\left(\frac{2a}{1 + a^2}\right) + \frac{1}{2}\cos^{-1}\left(\frac{1 - a^2}{1 + a^2}\right)\right]$

20. Prove that $\tan^{-1}\left(\frac{x}{y}\right) - \tan^{-1}\left(\frac{x - y}{x + y}\right) = 1$

IV. Answer any 4 of the following **$(4 \times 5 = 20)$**

21. Solve: $\tan^{-1}(x - 1) + \tan^{-1} x + \tan^{-1}(x + 1) = \tan^{-1}(3x)$

22. Solve for x: $\tan^{-1}\left[\frac{1 + x}{1 - x}\right] = \frac{\pi}{4} + \tan^{-1} x,\ 0 < x < 1.$

23. Find the value of $\sin^{-1}(-1) + \cos^{-1}\left(\frac{1}{2}\right) + \cot^{-1}(2)$

24. Find the domain of

    (i) $f(x) = \sin^{-1}\left(\frac{|x| - 2}{3}\right) + \cos^{-1}\left(\frac{1 - |x|}{4}\right)$

    (ii) $g(x) = \sin^{-1} x + \cos^{-1} x$

25. If $a_1, a_2, a_3 \dots a_n$ is an arithmetic progression with common difference 'd' prove that

$$\tan\left[\tan^{-1}\left(\frac{d}{1 + a_1 a_2}\right) + \tan^{-1}\left(\frac{d}{1 + a_2 a_3}\right) + \dots \left[+ \tan^{-1}\left(\frac{d}{1 + a_n a_{n-1}}\right)\right]\right] = \frac{a_n - a_1}{1 + a_1 a_n}$$

CHAPTER 5. TWO DIMENSIONAL ANALYTICAL GEOMETRY - II

UNIT TEST: I (Ex 5.1)

Time: 45 min **Marks: 25**

I. Answer the following **$(3 \times 2 = 6)$**

1. Find the equation of the circle with centre $(2, -1)$ and passing through the point $(3, 6)$ in standard form.
2. The line $3x + 4y - 12 = 0$ meets the co-ordinate axes at A and B. Find the equation of the circle drawn on AB as diameter.
3. If $y = 2\sqrt{2}x + c$ is a tangent to the circle $x^2 + y^2 = 16$. Find the value of c.

II. Answer the following **$(3 \times 3 = 9)$**

4. Find the centre and radius of the circle $3x^2 + (a+1)y^2 + 6x - 9y + a + 4 = 0$.
5. Find the equation of the tangent and normal to the circle $x^2 + y^2 - 6x + 6y - 8 = 0$ at $(2, 2)$.
6. A circle of perimeter 8π units has two of its diameters along the lines $x + y = 5$ and $x - y = 1$. Find the equation of the circle.

III. Answer the following **$(2 \times 5 = 10)$**

7. Find the equation of the circle passing through the points $(1, 1)$, $(2, -1)$ and $(3, 2)$.
8. Find the equation of the circle describe on the chord $3x + y + 5 = 0$ of the circle $x^2 + y^2 = 16$ as diameter.

UNIT TEST: II
(Ex 5.2, 5.3, 5.4)

Time: 45 min **Marks: 25**

I. Answer the following **$(3 \times 2 = 6)$**

1. Identify the type of conic
   (i) $3x^2 + 3y^2 - 4x + 3y + 10 = 0$
   (ii) $x^2 - 2y = x + 3$
2. Find the vertices, Foci for the hyperbola $9x^2 - 16y^2 = 144$.
3. Find the equation of the parabola given vertex $(1, -2)$ and Focus $(4, -2)$

II. Answer the following **$(3 \times 3 = 9)$**

4. Find the equation of the ellipse given length of latus rectum 4, distance between foci $4\sqrt{2}$ and major axis as y axis.
5. Show that the line $x - y + 4 = 0$ is tangent to the ellipse $x^2 + 3y^2 = 12$. Also find the co-ordinates of the point of contact.
6. Find the vertex, focus, equation of the directrix and length of latus rectum of $x^2 - 2x + 8y + 17 = 0$.

III. Answer the following **$(2 \times 5 = 10)$**

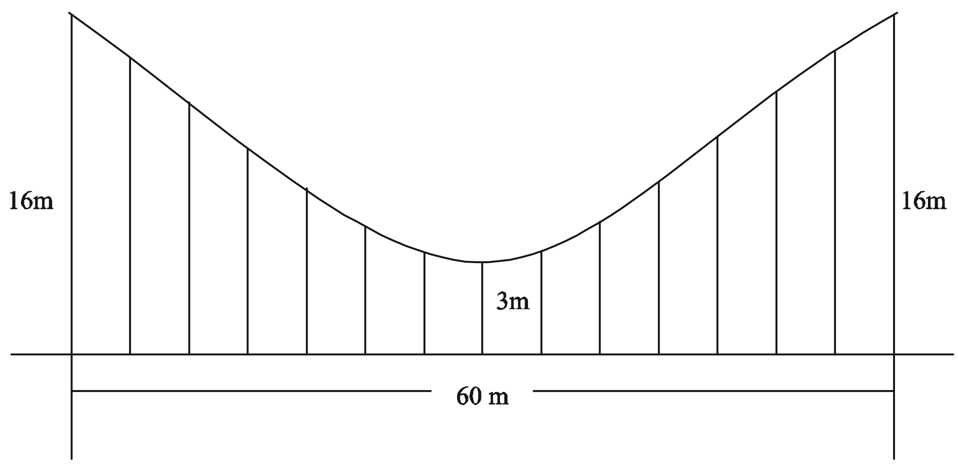
7. Identify the type of conic and find centre, foci, vertices and directrices of $11x^2 - 25y^2 - 44x + 50y - 256 = 0$.
8. Find the equations of tangent and normal to the ellipse $x^2 + y^2 = 32$ when $\theta = \frac{\pi}{4}$

UNIT TEST: III

Time: 45 min **Marks: 25**

I. Answer the following **$(5 \times 5 = 25)$**

1. On lighting a rocket cracker it gets projected in a parabolic path and reachers a maximum height of 4 m when it is 6 m away from the point of projection. Finally it reaches the ground 12 m away from the starting point. Find the angle of projection.
2. Parabolic cable of a 60 m portion of the road bed of a suspension bridge are positioned as shown below. Vertical cables are to be spaced every 6 m along this portion of the road bed. Calculate the lengths of first two of these vertical cables from the vertex.



3. A semi-elliptical arch way over a one way road has a height of 3 m and a width of 12 m. The truck has a width of 3 m and a height of 2.7 m. Will the truck clear the opening of the arch way.
4. A rod of length 1.2 m moves with its ends always touching the co-ordinate axes. The 1ocus of a point P on the rod, which is 0.3 m from the end in contact with x axis is an ellipse. Find the eccentricity.
5. Points A and B are 10 km apart and it is determined from the sound of an explosion heard at those points at different times than the location of the explosion is 6 km closer to A than B show that the location of the explosion is restricted a particular curve and find an equation of it.

CHAPTER TEST 5: TWO DIMENSIONAL ANALYTICAL GEOMETRY

Time: 1.30 hrs **Marks: 50**

I. Choose the correct answer **$(10 \times 1 = 10)$**

1. The circle $x^2 + y^2 = 4x + 8y + 5$ intersects the line $3x - 4y = m$ at two distinct points if

   (A) $15 < m < 65$ (B) $35 < m < 85$
   (C) $-85 < m < -35$ (D) $-35 < m < 15$

2. Which of the following statements are true?

   (i) If (x_1, y_1) is a point outside the circle then both tangents are real.

   (ii) If (x_1, y_1) is a point inside the circle then both tangents are imaginary.

   (iii) If (x_1, y_1) is a point on the circle then both the tangents coincide.

   (A) (i) only (B) (ii), (iii) only (C) (iii) only (D) All above

3. The radius of the circle $3x^2 + by^2 + 4bx - 6by + b^2 = 0$ is

   (A) 1 (B) 3 (C) $\sqrt{10}$ (D) $\sqrt{11}$

4. If $x + y = k$ is a normal to the parabola $y^2 = 12x$, then the value of k is

   (A) 9 (B) 1 (C) -1 (D) 3

5. The ellipse $E_1 : \frac{x^2}{9} + \frac{y^2}{4} = 1$ is inscribed in a rectangle R whose sides are parallel to the coordinate axes. Another ellipse E_2 passing through the point (0, 4) circumscribes the rectangle R. The eccentricity of the ellipse is

   (A) $\frac{\sqrt{2}}{2}$ (B) $\frac{\sqrt{3}}{2}$ (C) $\frac{1}{2}$ (D) $\frac{3}{4}$

6. The directrix of the circle is at

   (A) 1 (B) 0 (C) infinity (D) any real number

7. The eccentricity of the ellipse $(x-3)^2 + (y-4)^2 = \frac{y^2}{9}$ is

   (A) $\frac{1}{\sqrt{3}}$ (B) $\frac{1}{3}$ (C) $\frac{\sqrt{3}}{2}$ (D) $\frac{1}{3\sqrt{2}}$

8. The values of m for which the line $y = mx + 2\sqrt{5}$ touches the hyperbola $16x^2 - 9y^2 = 144$ are the roots of $x^2 - (a+b)x - 4 = 0$, then the value of $(a+b)$ is

   (A) 2 (B) 4 (C) 0 (D) -2

9. Area of the greatest rectangle inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

   (A) $2ab$ (B) ab (C) $\sqrt{ab}$ (D) $\frac{a}{b}$

10. The straight line $2x - y + c = 0$ is a tangent to the ellipse $4x^2 + 8y^2 = 32$ if c is

(A) $\pm 2\sqrt{3}$ (B) ± 6 (C) 36 (D) ± 4

II. Answer any 4 of the following (Q.No: 15 is Compulsory) **$(4 \times 2 = 8)$**

11. Obtain the equation of the circle for which (3, 4) and (2, – 7) are the ends of a diameter.
12. Find the equation of the ellipse with foci $(\pm 2, 0)$ vertices $(\pm 3, 0)$.
13. Find the equation of the tangent to the parabola $y^2 = 16x$ perpendicular to $2x + 2y + 3 = 0$.
14. Prove that the length of the latus rectum of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $\frac{2b^2}{a}$.

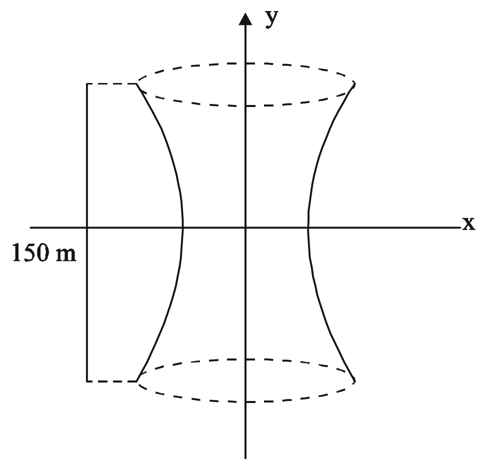
III. Answer any 3 of the following (Q.No: 18 is Compulsory) **$(3 \times 3 = 9)$**

15. A line $3x + 4y + 10 = 0$ cuts a chord of length 6 units on a circle with centre of the circle (2, 1). Find the equation of the circle in general form.
16. Find the area of Quadrilateral formed with foci of the hyperbolas $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$
17. The parabolic communication antenna has a focus at 2 m distance from the vertex of the antenna. Find the width of the antenna 3 m from the vertex.
18. If the normal at the point 't_1' on the parabola $y^2 = 4ax$ meets the parabola again at the point 't_2', then prove that $t_2 = -\left(t_1 + \frac{2}{t_1}\right)$.

IV. Answer any 4 of the following **$(5 \times 5 = 25)$**

19. Find the equation of the circle passing through the three points (1, 2), (3, – 4) and (5, – 6).
20. Identify the type of conic and find centre, foci, vertices and directrices of $18x^2 + 12y^2 - 144x + 48y + 120 = 0$.
21. Show that the absolute value of difference of the focal distance of any P on the hyperbola is the length of its transverse axis.
22. Find the equations of the tangent and normal to hyperbola $12x^2 - 9y^2 = 108$ at $\theta = \frac{\pi}{3}$.
23. A bride has a parabolic such that is 10 m high in the centre and 30 m wide at the bottom. Find the height of the arch 6 m from the centre on either sides.
24. A tunnel through a mountain for a four lane highway is to have a elliptical opening. The total width of the highway (not the opening) is to be 16 m, and the height at the edges of the road must be sufficient for a truck 4 m high to clear if the highest point of the opening is to be 5 m approximately. How wide must the opening be?

25. Cross section of a nuclear cooling tower is in the ellipse of a hyperbola with equation $\frac{x^2}{30^2} - \frac{y^2}{44^2} = 1$. The tower is 150 m fall and the distance from the top of the tower at the centre of the hyperbola is half the distance from the base of the tower to the centre of the hyperbola. Find the diameter of the top and base of the tower.



CHAPTER 6. APPLICATIONS OF VECTOR ALGEBRA

UNIT TEST: I (Ex 6.1, 6.2, 6.3)

Time: 45 min **Marks: 25**

I. Answer the following **$(3 \times 2 = 6)$**

1. Find the magnitude and direction cosines of the torque of a force represented by $3\vec{i} + 4\vec{j} - 5\vec{k}$ about the point with position vector $2\vec{i} - 3\vec{j} + 4\vec{k}$ acting through a point whose position vector is $4\vec{i} + 2\vec{j} - 3\vec{k}$.
2. With usual notations in any triangle ABC prove that $a = b\cos C + \cos B$ using vector method.
3. If $2\vec{i} - \vec{j} + 3\vec{k}$, $3\vec{i} + 2\vec{j} + \vec{k}$, $\vec{i} + m\vec{j} + 4\vec{k}$ are coplanar, find the value of m.

II. Answer the following **$(3 \times 3 = 9)$**

4. Prove that $[\vec{a} - \vec{b}\ \ \vec{b} - \vec{c}\ \ \vec{c} - \vec{a}] = 0$.
5. Prove by vector method that an angle in a semi circle is a right angle.
6. Let $\vec{a}, \vec{b}, \vec{c}$ be three non-zero vectors such that $\vec{c}$ is a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$. If the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{6}$. Show that $[\vec{a}\ \vec{b}\ \vec{c}]^2 = \frac{1}{4}|\vec{a}|^2\,|\vec{b}|^2$.

III. Answer the following **$(2 \times 5 = 10)$**

7. Prove by vector method $\sin(\alpha + \beta) = \sin\alpha \times \cos\beta + \cos\alpha \sin\beta$.
8. If $\vec{a} = 2\vec{i} + 3\vec{j} - \vec{k}$, $\vec{b} = 3\vec{i} + 5\vec{j} + 2\vec{k}$, $\vec{c} = -\vec{i} - 2\vec{j} + 3\vec{k}$ verify that

   (i) $(\vec{a} \times \vec{b}) \times \vec{c} = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{b} \cdot \vec{c})\vec{a}$

   (ii) $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$

UNIT TEST: II (Ex 6.4, 6.5)

Time: 45 min **Marks: 25**

I. Answer the following **$(3 \times 2 = 6)$**

1. Find the parametric form of vector equation and cartesian equations of the straight line passing through the point $(-2, 3, 4)$ and parallel to the straight line $\frac{x-1}{-4} = \frac{y+3}{5} = \frac{8-z}{6}$.
2. Find the angle between the lines $\frac{x+4}{3} = \frac{y-7}{4} = \frac{z+5}{5}$, $\vec{r} = 4\vec{k} + t(2\vec{i} + \vec{j} + \vec{k})$
3. If two lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-m}{2} = z$ intersect at a point, find the value of 'm'.

II. Answer the following **$(3 \times 3 = 9)$**

4. If the straight lines $\frac{x-5}{5m+2} = \frac{2-y}{5} = \frac{1-z}{-1}$ and $x = \frac{2y+1}{4m} = \frac{1-z}{-3}$ are perpendicular to each other, then find the value of m.
5. Find the points where the straight line passes through (6, 7, 4) and (8, 4, 9) cuts the xz and yz planes.
6. Show that the straight lines $x + 1 = 2y = -12z$ and $x = y + 2 = 6z - 6$ are skew lines and hence find the shortest distance between them.

III. Answer the following **$(2 \times 5 = 10)$**

7. Show that the lines $\frac{x-3}{3} = \frac{y-3}{-1}$, $z - 1 = 0$ and $\frac{x-6}{2} = \frac{z-1}{3}$, $y - 2 = 0$ intersect. Also find the point of intersection.
8. Find the co-ordinates of the foot of the perpendicular drawn from the point $(-1, 2, 3)$ to the straight line $\vec{r} = (\vec{i} - 4\vec{j} + 3\vec{k}) + t(2\vec{i} + 3\vec{j} + \vec{k})$. Also find the shortest distance from the point to the straight line.

UNIT TEST: III
(Ex 6.6, 6.7, 6.8, 6.9)

Time: 45 min **Marks: 25**

I. Answer the following $(2 \times 2 = 4)$

1. Find the vector and cartesian form of the equations of a plane which is at a distance of 12 units from the origin and perpendicular to $6\vec{i} + 2\vec{j} - 3\vec{k}$.
2. Find the angle between the planes $\vec{r} \cdot (\vec{i} + \vec{j} - 2\vec{k}) = 3$ and $2x - 2y + z = 2$.

II. Answer the following $(2 \times 3 = 6)$

3. Find the equation of the plane which passes through the point $(3, 4, -1)$ and is parallel to the plane $2x - 3y + 5z + 7 = 0$. Also find the distance between the two planes.
4. If the straight lines $\dfrac{x-1}{1} = \dfrac{y-2}{2} = \dfrac{z-3}{m^2}$ and $\dfrac{x-3}{1} = \dfrac{y-2}{m^2} = \dfrac{z-1}{2}$ are coplanar, find the distinct real values of m.

III. Answer the following $(3 \times 5 = 15)$

5. Find the parametric form of vector equation and cartesian equations of the plane containing the line $\vec{r} = (\vec{i} - \vec{j} + 3\vec{k}) + t(2\vec{i} - \vec{j} + 4\vec{k})$ and perpendicular to plane $\vec{r} \cdot (\vec{i} + 2\vec{j} + \vec{k}) = 8$
6. Find the parametric form of vector equation and cartesian equation of the plane passing through the points (2, 2, 1), (9, 3, 6) and perpendicular to the plane $2x + 6y + 6z = 9$.
7. Find the equation of the plane passing through the line of intersection of the planes $\vec{r} \cdot (2\vec{i} - 7\vec{j} + 4\vec{k}) = 3$ and $3x - 5y + 4z + 11 = 0$ and the point $(-2, 1, 3)$.

CHAPTER TEST 6: APPLICATIONS OF VECTOR ALGEBRA

Time: 1.30 hrs **Marks: 50**

I. Choose the correct answer

$(10 \times 1 = 10)$

1. If $\vec{a}$ and $\vec{b}$ are parallel vectors, then $[\vec{a}\ \vec{b}\ \vec{c}]$ is equal to

   (A) 2 (B) -1 (C) 1 (D) 0

2. For any three vectors $\vec{a}, \vec{b}, \vec{c}$ we have $\vec{a} \times (\vec{b} \times \vec{c}) + \vec{b} \times (\vec{c} \times \vec{a}) + \vec{c} \times (\vec{a} \times \vec{b}) =$

   (A) 0 (B) 1 (C) -1 (D) any real number

3. If $\vec{a} = \vec{i} + \vec{j} + \vec{k}, \vec{b} = \vec{i} + \vec{j}, \vec{c} = \vec{i}$ and $(\vec{a} \times (\vec{b}) \times \vec{c} = \lambda\, \vec{a} + \mu\, \vec{b},$ then the value of $\lambda + \mu$ is

   (A) 0 (B) 1 (C) 6 (D) 3

4. If $[\vec{a} \times \vec{b}\ \vec{b} \times \vec{c}\ \vec{c} \times \vec{a}] = 64$ then $[\vec{a}\ \vec{b}\ \vec{c}]$ is

   (A) 32 (B) 128 (C) 0 (D) 8

5. If $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \times \vec{b}) \times \vec{c}$ where $\vec{a}, \vec{b}, \vec{c}$ are any three vectors such that $\vec{b}, \vec{c} \neq 0$ and $\vec{a} \cdot \vec{b} \neq 0$, then $\vec{a}$ and $\vec{c}$ are

   (A) Inclined at an angle $\frac{\pi}{3}$ (B) Inclined at an angle $\frac{\pi}{6}$

   (C) Parallel (D) Perpendicular

6. If the line $\vec{r} = \vec{a} + t\,\vec{b}$ lies in the plane $\vec{r} \cdot \vec{n}$ then

   (A) $\vec{a} \cdot \vec{n} = d$ and $\vec{b} \cdot \vec{n} = 0$ (B) $\vec{a} \cdot \vec{b} = \vec{n}$ and $\vec{a} \cdot \vec{b} = 0$

   (C) $\vec{a} \cdot \vec{n} = d$ and $\vec{b} \cdot \vec{n} \neq 0$ (D) $\vec{a} \cdot \vec{b} = d$ and $\vec{a} \cdot \vec{n} = 0$

7. If the line $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$ intersect, then k is equal to

   (A) -1 (B) 2/9 (C) 9/2 (D) 0

8. Distance from the origin to the plane $3x - 6y + 2z + 7 = 0$ is

   (A) 1 (B) 2 (C) 3 (D) 0

9. If the direction cosines of a line are $\frac{1}{c}, \frac{1}{c}, \frac{1}{c}$ then

   (A) $c = \pm 3$ (B) $c = \pm\sqrt{3}$ (C) $c > 0$ (D) $0 < c < 1$

10. If the planes $\vec{r} \cdot (2\,\vec{i} - \lambda\,\vec{j} + \vec{k}) = 3$ and $\vec{r} \cdot (4\,\vec{i} + \vec{j} - \mu\, k) = 5$ are parallel then the value of λ and μ are

    (A) $\frac{1}{2}, -2$ (B) $-\frac{1}{2}, 2$ (C) $\frac{-1}{2}, -2$ (D) $\frac{1}{2}, 2$

II. Answer any 4 of the following (Q.No: 16 is Compulsory) **$(4 \times 2 = 8)$**

11. The constant forces $2\vec{i} - 5\vec{j} + 6\vec{k}, -\vec{i} + 2\vec{j} - \vec{k}$ and $2\vec{i} + 7\vec{j}$ act on a particle which is displaced from position $4\vec{i} - 3\vec{j} - 2\vec{k}$ to position $6\vec{i} + \vec{j} - 3\vec{k}$. Find the work done.

12. If the vectors $a\vec{i} + a\vec{j} + c\vec{k}, \vec{i} + \vec{k}$ and $c\vec{i} + c\vec{j} + b\vec{k}$ are coplanar, prove that c is the geometric mean of a and b.

13. Show that the lines $\frac{x-1}{4} = \frac{2-y}{6} = \frac{z-4}{12}$ and $\frac{x-3}{-2} = \frac{y-3}{3} = \frac{5-z}{6}$ are parallel.

14. Find the direction cosines and length of the perpendicular from the origin to the plane $\vec{r} \cdot (3\vec{i} - 4\vec{j} + 12\vec{k}) = 5$.

15. Find the angle between the line $\vec{r} = (2\vec{i} - \vec{j} + \vec{k}) + t(\vec{i} + 2\vec{j} - 2\vec{k})$ and the plane $\vec{r} \cdot (6\vec{i} + 3\vec{j} + 2\vec{k}) = 8$

16. If any triangle ABC prove that $b = a\cos C + c\cos A$ using vector method.

III. Answer any 4 of the following (Q.No: 22 is Compulsory) **$(4 \times 3 = 12)$**

17. Angle in a semicircle is a right angle. Prove by vector method.

18. If the straight lines $\frac{x-5}{2+5m} = \frac{2-y}{5} = \frac{1-z}{-1}$ and $x = \frac{2y+1}{4m} = \frac{1-z}{-3}$ are perpendicular to each other, then find the value of m.

19. Prove that $[\vec{a} + \vec{b}\ \ \vec{b} + \vec{c}\ \ \vec{c} + \vec{a}] = 2[\vec{a}\ \vec{b}\ \vec{c}]$

20. Find the points where the straight line passes through (6, 7, 4) and (8, 4, 9) cuts the xz and yz planes.

21. Find the co-ordinates of the point where the straight line $\vec{r} = (2\vec{i} - \vec{j} + 2\vec{k}) + t(3\vec{i} + 4\vec{j} + 2\vec{k})$ intersects the plane $x - y + z - 5 = 0$.

22. Show that the two lines $\vec{r} = (\vec{i} - \vec{j}) + t(2\vec{i} + \vec{k})$ and $\vec{r} = (2\vec{i} - \vec{j}) + s(\vec{i} + \vec{j} - \vec{k})$ are skew lines and find the distance between them.

IV. Answer the following **$(4 \times 5 = 20)$**

23. (a) Prove by vector method $\cos(A + B) = \cos A\cos B - \sin A\sin B$

(Or)

(b) Show that the lines $\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{3}$ and $\frac{x-1}{-3} = \frac{y-4}{2} = \frac{z-5}{1}$ are coplanar. Also find the plane containing these lines.

24. (a) If $\vec{a} = \vec{i} + \vec{j} + \vec{k}, \vec{b} = 2\vec{i} + \vec{k}, \vec{c} = 2\vec{i} + \vec{j} + \vec{k}, \vec{d} = \vec{i} + \vec{j} + 2\vec{k}$ verify $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = [\vec{a}\ \vec{b}\ \vec{d}]\vec{c} - [\vec{a}\ \vec{b}\ \vec{c}]\vec{d}$

(Or)

(b) Show that the lines $\frac{x-3}{3} = \frac{y-3}{-1}, z-1=0$ and $\frac{x-6}{2} = \frac{z-1}{3}, y-2=0$ intersect. Also find the point of intersection.

25. (a) Find the parametric form of vector equation and cartesian equation of the plane passing through the points (2, 2, 1) and (9, 3, 6) and perpendicular to the plane $2x+6y+6z=9$.

(Or)

(b) Find the non-parametric form of vector equation and cartesian equations of the plane $\vec{r} = (6\vec{i} - \vec{j} + \vec{k}) + s(-\vec{i} + 2\vec{j} + \vec{k}) + t(-5\vec{i} - 4\vec{j} - 5\vec{k})$.

26. (a) Find the root of the perpendicular drawn from the point (5, 4, 2) to the line $\frac{x+1}{2} = \frac{y-3}{3} = \frac{z-1}{-1}$. Also find the equation of the perpendicular.

(Or)

(b) Altitudes of a triangle are concurrent prove by vector method.

CHAPTER 7. APPLICATIONS OF DIFFERENTIAL CALCULUS

UNIT TEST: 1 (Ex 7.1, 7.2)

I. Answer the following **$(2\times 2=4)$**

1. If the volume of a cube of side length x is $v=x^3$. Find the rate of change of the volume with respect to x when $x=5$ units.
2. Find the angle of intersection of the curve $y=\sin x$ with the positive x-axis.

II. Answer the following **$(2\times 3=6)$**

3. A camera is accidentally knocked off an edge of a cliff 400 ft height. The camera falls a distance of $s=16t^2$ in t seconds.
   (i) How long does the camera fall before it hits the ground.
   (ii) What is the average velocity with which the camera falls during the last 2 seconds?
   (iii) What is the instantaneous velocity of the camera when it hits the ground.
4. Show that the two curves $x^2-y^2=r^2$ and $xy=c^2$ where c, r are constants cut orthogonally.

III. Answer the following **$(3\times 5=15)$**

5. A conical water tank with vertex down of 12 metres height has a radius of 5 metres at the top. If water flows into the tank at the rate of 10 cubic m/min. how fast is the depth of the water increases when the water is 8 m deep?
6. Find the equation of the tangent and normal to the Lissajous curve given by $x=2\cos 3t$ and $y=3\sin 2t,\ t\in R$.
7. If the curves $ax^2+by^2=1$ and $cx^2+dy^2=1$ intersect each other orthogonally then prove that $\frac{1}{a}-\frac{1}{b}=\frac{1}{c}-\frac{1}{d}$.

UNIT TEST: II (Ex 7.3, 7.4, 7.5)

I. Answer the following **($2 \times 3 = 6$)**

1. Verify Rolle's theorem is applicable or not to the function $f(x) = x - 2\log x, x \in [2, 7]$
2. Prove using mean value theorem that $|\sin \alpha - \sin \beta| \le |\alpha - \beta|$, $\alpha\, \beta \in R$.
3. Evaluate $\lim\limits_{x \to 0} \left(\frac{\sin mx}{x}\right)$

II. Answer the following **($3 \times 3 = 9$)**

4. Using the Lagrange's mean value theorem determine the value of x at which the tangent is parallel to the secant line at the end points of the given interval:
$$f(x) = x^3 - 3x + 2, x \in [-2, 2]$$
5. Write the Maclaurin series expansion of the function "$\sin x$".
6. Evaluate: $\lim\limits_{x \to 0} \left(\frac{1}{\sin x} - \frac{1}{x}\right)$

III. Answer the following **($2 \times 5 = 10$)**

7. Expand $\tan x$ in ascending powers of x upto 5^{th} power for $\frac{-\pi}{2} < x < \frac{\pi}{2}$.
8. Evaluate $\lim\limits_{x \to \pi/2} (\sin x)^{\tan x}$

UNIT TEST: III (Ex 7.6, 7.7, 7.8, 7.9)

I. Answer the following **$(2 \times 2 = 4)$**

1. Find the critical points of the function $f(x) = 6x^{4/3} - 3x^{1/3}$; $[-1, 1]$
2. Prove that the function $f(x) = x - \sin x$ is increasingly on the real line. Also discuss for the existence of local extrema.

II. Answer the following **$(2 \times 3 = 6)$**

3. Find the absolute maximum and absolute minimum values of the function

$$f(x) = 2x^3 + 3x^2 - 12x \text{ on } [-3, 2]$$

4. Find intervals of concavity and points of inflexion for the function

$$f(x) = \sin x + \cos x, 0 < x < 2\pi$$

III. Answer the following **$(3 \times 5 = 15)$**

5. For the function $f(x) = 4x^3 + 3x^2 - 6x + 1$ find the intervals of monotonicity, local extrema, intervals of concavity and point of inflection $f(x) = 4x^3 + 3x^2 - 6x + 1$
6. Find the dimensions of the largest rectangle that can be inscribed in a circle of radius 'r' cm.
7. Sketch the graphs of the function $y = \frac{-1}{3}(x^3 - 3x + 2)$.